Regulating network charges in an upstream monopoly market that supplies a downstream retail duopoly: A potential framework for NBN pricing

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Abstract

I model an industry in which an upstream monopoly sells network services to a midstream duopoly, which sells the services on to households. This framework examines the welfare implications of varying the regulatory goals applied to the Australian National Broadband Network. In the model the monopolist’s price is regulated and the duopoly maximises profits by limiting quantities sold. I find the total surplus optimising regulated price is strongly determined by the relative priorities given to consumer surplus, monopolist profits and duopoly profits. Duopoly profits and consumer surplus increase together, but move in the opposite direction to monopolist profits, raising a policy dilemma especially where monopoly networks are owned by the agent which sets regulatory policy.

1 Introduction

Many economically important networks approximate natural monopolies, due to scale efficiencies and the costs of translating content across separate connected networks. However horizontal integration of one link in a supply chain does not imply vertical integration of the industry - a monopoly network may transact upstream with many suppliers and downstream with many customers. Such a network, if it is a profit-seeking entity which charges per unit for its services, has an incentive to exploit its position to secure monopoly rents; it will sell fewer units at a higher price than the industry would under perfect competition. Governments seeking to maximise economic welfare may regulate monopoly production quantities or price levels, trading off some monopoly rents to increase consumer and total surplus. This trade off applies to the National Broadband Network (NBN) being built by the Australian government, and operating as a monopoly with regulated pricing and protection from competition. Regulatory solutions for monopolies operating in otherwise competitive markets are well known, but trading off surpluses is complicated in the presence of other imperfectly competitive layers in the industrial structure.

Haucap and Klein [2012] consider investment incentives in such a market, where an upstream monopoly network provides services to a downstream
They focus on investments in service quality, and assume the network charge to access the monopoly’s services is exogenously regulated. This paper demonstrates that the optimal network charge can be determined endogenously, if the preferences of the price regulator are known. The regulator assigns separate importance to consumer surplus, the profits of the downstream duopoly, and the profits of the monopoly, and sets a network charge accordingly. The downstream firms adjust their quantities sold in response, in Cournot fashion. This paper finds the efficient network charge depends strongly on the regulator’s policy, expressed as weights applied to the profits and consumer surplus. Total welfare is not a zero sum allocation game between consumers, the network and the downstream firms; rather prioritising network profits detracts from total welfare. Also, downstream producer profits and consumer surplus are not in rivalry, as they grow or shrink together in response to adjustment of the network charge. Setting or assessing the effectiveness of price regulation therefore requires an understanding of the policy settings, whether clearly defined or implicit.

These findings clarify the trade-offs faced by regulators, and have particular policy relevance where the operation or the privatisation of government-owned networks is considered. Rents extracted from government ownership of monopoly networks may be important fiscally, either as funds drawn from ongoing operation or to maximise (to investors) the value of a network being privatised. However such rents may detract from the overall economic contribution of the network, and the resulting distortion of consumer and midstream firm behaviour might have indirect effects on government revenue and expense. Regarding the NBN, downstream firms accessing the network have called for lower access pricing to increase quantities sold [Battersby, 1 October 2016], while government bodies are concerned that the future sale price of the network under privatisation will be too low to recoup the build cost. Then large and unanticipated costs will be borne by the national government.

This paper describes a simplified industrial structure similar to the NBN, where a price-regulated monopoly sells services to an unregulated duopoly. The industry’s profit-maximising responses to the regulation in terms of price, quantity and surpluses are derived, and the importance of policy priorities to achieving an efficient regulated price are discussed.

2 Model

I consider the case of a monopoly-owned network, access to which is purchased by an oligopoly of retailers and resold to households with some value added. The system is illustrated in Figure 1. There are two distinct prices to consider: the price at which the monopolist sells to the retailers ($p_N$), and the price (or prices) at which retailers sell the good to households ($p$ for one uniform price or $p_i$ for a nonuniform price charged by the $i$th firm, $i > 1$). The transaction between the monopolist and the retailers is regulated by setting the price, and the transaction between the oligopolistic retailers and households is unregulated. The regulated network price is announced, and the retailers respond by setting quantities to maximise profit. In this industry:

1. One monopolist owns the network, which is an unsubstitutable factor of production for the industry. The monopoly might exist though public
ownership, or a requirement for interoperability and interconnection, or by control of intellectual policy, or through management action, etc. The monopolist makes no decisions - it produces whatever quantity the retailers demand (the sum of quantities is \(q_N\)) and sells at the regulated price.

2. The regulator sets \(p_N\) to maximise some element of welfare. The regulator makes no attempt to directly control quantities, or retail prices.

3. A small number of competing retail firms (hereafter "retailers") buy the complete production of the network and sell it as an undifferentiated product to households. There is no threat of entry or possibility of merger, and retailers can individually control the quantity of network access (\(q_i\)) they sell. Retailers do not attempt to cooperate in setting quantities or prices, i.e. they refrain from forming a cartel. This paper considers the case where \(i = 2\).

4. Households select a particular firm’s product first by price and otherwise stochastically. The household’s decision to buy or not buy network access is expressed by a Marshallian linear demand function.

I consider the long run equilibrium, where the network owner has zero fixed costs and zero variable costs, earning profits purely by leasing access to the network. (More generally, \(p_N\) may be considered the average network access price net of average cost; the analysis is not affected.)

Retailer 1 and Retailer 2 buy the complete production of the network owner and sell it as an undifferentiated product to households. The retailers’ product is entirely supplied by the network, so \(q_N = q_1 + q_2\). The retailers’ costs consist of the network access price per unit quantity, as set by the regulator, plus a homogeneous variable cost \(c\). Retailers have zero fixed costs in the long run, so cost per retailer is \(C_i = q_i(p_N + c)\).

There are no regulatory or transaction costs except those embodied in \(p_N\) and \(c\).
Firms face linear inverse demand functions, as described by Singh and Vives [1984], thus

$$p_1 = \alpha_1 - \beta_1 q_1 - \gamma q_2$$

$$p_2 = \alpha_2 - \beta_2 q_2 - \gamma q_1$$

As the firms’ products are perfect substitutes the demand functions are identical,

$$\alpha_1 = \alpha_2 = \alpha$$

$$\beta_1 = \beta_2 = \gamma = \beta$$

Allowing only downward-sloping demand curves for ordinary goods and positive retail prices,

$$\alpha > 0$$

$$\beta > 0$$

Hence under the law of one price the uniform price charged to households by retailers for network access is

$$p_1 = p_2 = p = \alpha - \beta q_1 - \beta q_2$$  \hfill (1)

3 Quantities, profits and surpluses

Total surplus is the sum of the network owner’s profits ($Y_N$), the retailers’ profits ($Y_{PS}$), and consumer surplus ($Y_{CS}$), illustrated in Figure 2. Producer surplus is here defined as only the profits of the retailers; the network owner’s surplus is always considered separately. Producer surplus is set equal to the retailers’ profits by the assumption of no fixed costs for the retail firms [Perloff, 2014, p 262]. Total surplus, without applying any policy weightings, is

$$Y = Y_N + Y_{PS} + Y_{CS}$$

$$= \pi_N + \pi_1 + \pi_2 + Y_{CS}$$  \hfill (2)
Retailer and network profits are equal to their respective revenues minus total costs:

\[
\pi_1 = pq_1 - (p_N + c)q_1 \tag{3}
\]

\[
\pi_2 = pq_2 - (p_N + c)q_2 \tag{4}
\]

\[
\pi_N = p_N(q_1 + q_2) \tag{5}
\]

The Nash-Cournot equilibrium (see Appendix A) for retailers 1 and 2 corresponding to the regulated \( p_N \) is

\[
p = \frac{\alpha + 2(p_N + c)}{3} \tag{6}
\]

\[
q_1 = q_2 = \frac{(\alpha - c) - p_N}{3\beta} \tag{7}
\]

\[
q_N = q_1 + q_2 = \frac{2((\alpha - c) - p_N)}{3\beta} \tag{8}
\]

The total quantity sold declines linearly with the network price, as shown in Figure 3. Thus the solution to setting a quantity-maximising network price is trivial, i.e. set the price to zero.

At the Cournot prices and quantities, the profit of each retailer (see Appendix B) and the total producer surplus are

\[
\pi_1 = \pi_2 = \frac{1}{9\beta}((\alpha - c) - p_N)^2 \tag{9}
\]

\[
Y_{PS} = \pi_1 + \pi_2 = \frac{2}{9\beta}((\alpha - c) - p_N)^2 \tag{10}
\]

Network profit is the sum of network access charges, \( Y_N = p_Nq_N \), which at the Cournot equilibrium (see Appendix B) is

\[
Y_N = \frac{2p_N}{3\beta}((\alpha - c) - p_N) \tag{11}
\]
Given a linear demand curve, consumer surplus is a triangular area bounded vertically by $\alpha - p$ and horizontally by $q_N - 0$ (see Figure 4 and Appendix B).

$$Y_{CS} = \frac{1}{2}(\alpha - p)q_N$$

$$= \frac{2}{9\beta}((\alpha - c) - p_N)^2$$

$$= Y_{PS}$$  \hspace{1cm} (12)

The algebraic equivalence of the consumer surplus and the sum of retailer profits is a specific outcome of a duopoly with constant marginal costs that are equal for both firms if the demand curve is linear, which compete in Cournot fashion (see Appendices A and B). To avoid loss of generality, the equality of the surpluses will not be exploited in what follows.

To identify the network access price which maximises welfare given various policy settings, I apply weights (summing to one) to the different components of welfare, so Eq.2 becomes

$$Y = \chi Y_{PS} + \psi Y_{CS} + (1 - \chi - \psi)Y_N$$

$$= \frac{2\chi}{9\beta}((\alpha - c) - p_N)^2 + \frac{2\psi}{9\beta}((\alpha - c) - p_N)^2 + \frac{2(1 - \chi - \psi)p_N}{3\beta}((\alpha - c) - p_N)$$  \hspace{1cm} (13)

The first and second order derivatives of the total welfare function with respect to the network price are

$$\frac{\delta Y}{\delta p_N} = \frac{4\chi}{9\beta}(c + p_N - \alpha) + \frac{4\psi}{9\beta}(c + p_N - \alpha) + \frac{2(1 - \chi - \psi)p_N}{3\beta}((\alpha - c) - 2p_N)$$  \hspace{1cm} (14)

$$\frac{\delta^2 Y}{\delta p_N^2} = \frac{4}{9\beta}(\chi + \psi - 3(1 - \chi - \psi))$$

$$= \frac{4}{9\beta}(4(\chi + \psi) - 3)$$  \hspace{1cm} (15)

The sign of the second derivative identifies the critical point as a weighted-welfare maximum or minimum. Given $\beta$ is always positive, a maximum exists.
if
\[ \chi + \psi < \frac{3}{4}. \]

A welfare minimum exists if
\[ \chi + \psi > \frac{3}{4}, \]
in which case welfare can be maximised by setting \( p_N = 0 \). The policy implication of these results is discussed in Section 4 below.

The critical price (which may yield a maximum or minimum) is found by setting the first derivative to zero and solving for \( p_N \), giving
\[ p_N = \frac{\alpha - c}{2} \left( \frac{5(\chi + \psi) - 3}{4(\chi + \psi) - 3} \right) \quad (16) \]

The total quantity and welfare at the critical price (see Appendix B) are
\[ q_N = \frac{\alpha - c}{3\beta} \left( 2 - \frac{5(\chi + \psi) - 3}{4(\chi + \psi) - 3} \right) \]
\[ Y = \frac{2(\alpha - c)^2}{9\beta} \left( (\chi + \psi) - \frac{(5(\chi + \psi) - 3)^2}{16(\chi + \psi) - 12} \right) \quad (17) \]

Similarly, the elements of surplus at the critical price are
\[ Y_{CS} = \frac{2(\alpha - c)^2}{9\beta} \left( 1 - \frac{5(\chi + \psi) - 3}{8(\chi + \psi) - 6} \right)^2 \]
\[ Y_{PS} = \frac{2(\alpha - c)^2}{9\beta} \left( 1 - \frac{5(\chi + \psi) - 3}{8(\chi + \psi) - 6} \right)^2 \]
\[ Y_N = \frac{(\alpha - c)^2}{3\beta} \left( \frac{(5(\chi + \psi) - 3)}{(4(\chi + \psi) - 3)} \right) \left( 1 - \frac{5(\chi + \psi) - 3}{4(\chi + \psi) - 3} \right) \quad (19) \]

4 Policies

I initially constrain the range of the network price to be non-negative (i.e. subsidies are not possible, \( p_N > 0 \)). I assume the industry is potentially commercial, with households’ reserve demand price exceeding the marginal cost, \( \alpha > c \).

Inspection of the critical price equation (Eq.16) shows \( p_N \) is positive if either \( \chi + \psi > \frac{3}{4} \) (making the numerator and denominator positive) or \( \chi + \psi < \frac{3}{4} \) (making the numerator and denominator negative), yielding the applicable ranges (see Figure 5)
\[ 0 \leq \chi + \psi \leq 0.6 \quad \quad 0.75 < \chi + \psi \leq 1 \]

From Eq.17, the network quantity is non-negative in the range (see Figure 6)
\[ 0 < \chi + \psi \leq 0.75 \]

Given the conditions of positive prices and positive quantities, the welfare-optimising \( p_N \) is hence given by Eq.16 in the range \( 0 \leq \chi + \psi \leq 0.6 \), and is zero in the range \( 0.6 \leq \chi + \psi \leq 1 \). Table 1 shows optimised prices and the associated quantities and welfare for some outline policy options.
Table 1: Some weighted welfare optimisation policies

<table>
<thead>
<tr>
<th>Optimisation</th>
<th>χ</th>
<th>ψ</th>
<th>$p_N$</th>
<th>$q_N$</th>
<th>$Y^1_{PS}$</th>
<th>$Y^1_{CS}$</th>
<th>$Y^1_N$</th>
<th>Weighted $Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer+Producer+Network</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>0</td>
<td>$\frac{2(\alpha-c)}{3}$</td>
<td>$\frac{2(\alpha-c)^2}{9}$</td>
<td>$\frac{2(\alpha-c)^2}{9}$</td>
<td>0</td>
<td>$\frac{4(\alpha-c)^2}{27}$</td>
</tr>
<tr>
<td>Consumer alone</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\frac{2(\alpha-c)}{3}$</td>
<td>$\frac{2(\alpha-c)^2}{9}$</td>
<td>$\frac{2(\alpha-c)^2}{9}$</td>
<td>0</td>
<td>$\frac{2(\alpha-c)^2}{9}$</td>
</tr>
<tr>
<td>Producer alone</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$\frac{2(\alpha-c)}{3}$</td>
<td>$\frac{2(\alpha-c)^2}{9}$</td>
<td>$\frac{2(\alpha-c)^2}{9}$</td>
<td>0</td>
<td>$\frac{2(\alpha-c)^2}{9}$</td>
</tr>
<tr>
<td>Consumer+Producer</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$\frac{2(\alpha-c)}{3}$</td>
<td>$\frac{2(\alpha-c)^2}{9}$</td>
<td>$\frac{2(\alpha-c)^2}{9}$</td>
<td>0</td>
<td>$\frac{2(\alpha-c)^2}{9}$</td>
</tr>
<tr>
<td>Network alone</td>
<td>0</td>
<td>0</td>
<td>$\frac{\alpha-c}{2}$</td>
<td>$\frac{2(\alpha-c)}{3}$</td>
<td>$\frac{(\alpha-c)^2}{6}$</td>
<td>$\frac{(\alpha-c)^2}{6}$</td>
<td>0</td>
<td>$\frac{(\alpha-c)^2}{6}$</td>
</tr>
<tr>
<td>Consumer+Network</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\alpha-c}{4}$</td>
<td>$\frac{2(\alpha-c)}{27}$</td>
<td>$\frac{(\alpha-c)^2}{8}$</td>
<td>$\frac{(\alpha-c)^2}{8}$</td>
<td>0</td>
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<tr>
<td>Producer+Network</td>
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<td>0</td>
<td>$\frac{\alpha-c}{4}$</td>
<td>$\frac{2(\alpha-c)}{27}$</td>
<td>$\frac{(\alpha-c)^2}{8}$</td>
<td>$\frac{(\alpha-c)^2}{8}$</td>
<td>0</td>
<td>$\frac{(\alpha-c)^2}{8}$</td>
</tr>
</tbody>
</table>

1 Before weighting is applied
The second derivative of the welfare function (Eq.15) demonstrates that if the network profit is considered relatively unimportant (the weighting is set at less than 25%), the weighted-welfare maximising network price is always zero (Figure 7 shows an example). That is, if network profit has low policy importance, total welfare is best served by setting the network price at zero.\(^1\) However if the weighting is greater than 25%, a non-zero weighted-welfare maximising price always exists (Figure 8 shows an example). Such a policy balances the monopoly network’s profits against consumer and producer surpluses.

Allowing any network profit always decreases the quantity sold, the consumer surplus and the producer (i.e. retailers) surplus. Including some function for quantity maximisation to the weighted welfare function (for example to capture some external benefits of high network access) would increase downward pressure on the welfare maximising network price. To take an example from Table 1, the quantity difference between optimising for consumers alone and optimising equally for consumers and network profits is a 16% fall, with an almost 10% drop in the net policy outcome (weighted \(Y\)). The highest optimised network price results from applying zero weighting to consumer and producer surpluses.

In contrast, within the constraints of this model, there is no trade-off between consumer surplus and producer (i.e. the retail duopoly) profits. Any adjustment of the network price benefits or disadvantages consumers and producers in equal measure.

5 Network subsidy

If the requirement for a non-negative network price is removed, under some policies total weighted welfare is maximised by setting a negative network price, i.e.\(^1\)

\(^1\)Or at average cost if \(p_N\) is understood as price less a constant marginal cost, such as operating costs. This result requires revision for nonconstant marginal costs such as scale effects, amortisation or depreciation.
a subsidy. Eq.16 can then be applied across the extended range $0 \leq \chi + \psi \leq 0.75$, bounded only by the requirement for positive quantities. Figure 9 shows an example. Imagining the network as a government owned generator of external public good suggests conditions where a subsidy to maximise total welfare is politically possible. Taking as an example a government owned telecommunication network, the government may allow commercial services (telephone calls and internet access) to be delivered over the network, charging a toll which is less than the marginal cost of network operation and maintenance. The network might be financed in part from other government revenue, separate to network charges, if the government believed that investment of external revenue returned sufficiently large economic and public benefits. Then setting a subsidy level to maximise total welfare could be an attractive policy. Similar motivations see governments variously subsidise transport networks, water supply, electricity or heating fuel, which otherwise could be priced above cost.

The welfare-optimising subsidy is equal to the negative price, applying in the range $0.6 \leq \chi + \psi \leq 0.75$.

$$s_N = -p_N = \frac{c - \alpha}{2} \left( \frac{5(\chi + \psi) - 3}{4(\chi + \psi) - 3} \right)$$  \hspace{1cm} (20)

Welfare and quantity under the subsidy are as calculated by Eq.13 and Eq.17. Compared to a policy of setting a minimum price of zero (as shown in the first row of Table 1) with equally weighted surplus elements, the subsidy generates the extra quantity

$$\Delta q = \frac{\alpha - c}{3\beta} \left( 2 - \frac{5(\frac{1}{4} + \frac{1}{2}) - 3}{4(\frac{1}{3} + \frac{1}{4}) - 3} \right) = 2 \left( \frac{\alpha - c}{3\beta} \right)$$  \hspace{1cm} (21)

which is 50% higher than the unsubsidised quantity. The weighted total surplus
Figure 9: Sample weighted surplus maximised at $p_N < 0$

\[
\Delta Y = \frac{2(\alpha - c)^2}{9\beta} \left( \frac{2}{3} - \frac{5(\frac{2}{3})}{16(\frac{2}{3}) - 12} \right) - \frac{4(\alpha - c)^2}{27\beta}
\]

\[
= \frac{(\alpha - c)^2}{54\beta}
\]

which is an additional 12.5%.

6 Conclusion

Ordinarily regulation of monopoly pricing in otherwise perfectly competitive markets requires directly trading off monopoly rents for consumer surplus, but outcomes are less straightforward where industries mix monopolies with oligopolies. Regulatory action to increase consumer surplus can collateral increase oligopoly profits. Maximising the total welfare may require the monopoly to operate at a deficit. Optimal regulation requires explicit quantification of the relative preference given to consumer, monopoly and oligopoly value, and the allocations, \textit{ceteris paribus}, directly determine the optimal regulated price.

Government ownership of the monopoly complicates the regulatory task, as the simple economic benefit of the network can be greatest when its direct fiscal contribution is lowest. Considering the fiscal effects (through taxation) of producer and consumer surplus, and the public cost of operating and subsidising the network, within the model would improve its usefulness to policy makers, as might consideration of the external benefits and costs of network utilisation.

Extending the model from a duopoly of retailers to an oligopoly would also increase its usefulness. Not only would it be applicable in more situations (for example, retail broadband markets are typically served by between one and
four dominant firms), but could inform how antitrust policy applied to retailers affects welfare outcomes.

This paper has made the simplifying assumption of zero marginal costs for network access. Real networks have some cost to build, taken either as a lump sum up front or amortised over time, and such costs will be affected by the network extent and quality. Increased quality and reach should increase the network’s value to users, but it is reasonable to expect that the willingness to pay will reach points of diminishing returns. The analysis could be extended beyond this paper’s assumptions to consider increasing then decreasing returns (as price minus marginal cost) against increasing quantities.

The Australian National Broadband Network is an interesting application. The Australian Government intends the network to “deliver economic and social benefits for all Australians” by maximising access at affordable prices, but at least cost to taxpayers [Shareholder Ministers, 2016]. The incompatibility of these expectations means regulators must elicit or infer the preferences of policy makers to constrain NBN prices effectively.

Appendix A - Solution of the Cournot-Nash equilibrium for two retailers

In Eq.3, substitute Eq.1 for \( p \):

\[
\pi_1 = (\alpha - \beta q_1 - \beta q_2)q_1 - (p_N + c)q_1. \tag{23}
\]

Retailer 1 takes \( q_2 \) as given, and selects a production quantity to maximise profit accordingly.

\[
\frac{\delta \pi_1}{\delta q_1} = \alpha - 2\beta q_1 - \beta q_2 - p_N - c \tag{24}
\]

The second derivative is

\[
\frac{\delta^2 \pi_1}{\delta q_1^2} = -2\beta \tag{25}
\]

As \( \beta \) is always positive, the second derivative is always negative and thus always identifies a maximum. Solving Eq.24 locates the maximum at

\[
q_1 = \frac{\alpha - p_N - c}{2\beta} - \frac{q_2}{2} \tag{26}
\]

By symmetry (as the retailers are identical),

\[
q_2 = \frac{\alpha - p_N - c}{2\beta} - \frac{q_1}{2} \tag{27}
\]

Substituting Eq.27 into Eq.26 yields Eq.7:

\[
q_1 = \frac{\alpha - p_N - c}{2\beta} - \frac{\alpha - p_N - c}{4\beta} + \frac{q_1}{4}
\]

\[
= \frac{\alpha - p_N - c}{3\beta}
\]

Firm 2’s quantity is identical by symmetry, and Eq.8 is the sum of these quantities. Substituting these into the inverse demand function (Eq.1) generates Eq.6 for price.
Appendix B - Elements of surplus

Retailer 1 and retailer 2 have the same profit, found by substituting Eq.6 and Eq.7 into the firms’ profit function Eq.3.

\[
\pi_1 = \left( \frac{\alpha}{3} + \frac{2(p_N + c)}{3} \right) \left( \frac{\alpha - p_N - c}{3\beta} \right) - (p_N + c) \frac{\alpha - p_N - c}{3\beta} \\
= \frac{1}{9\beta} \left( \alpha^2 - 2\alpha p_N - 2\alpha c + p_N^2 + c^2 + 2p_Nc \right) \\
= \frac{1}{9\beta} \left( \alpha^2 - 2\alpha c + c^2 + 2p_N(c - \alpha) + p_N^2 \right) \\
= \frac{1}{9\beta} \left( (\alpha - c)^2 - 2p_N(\alpha - c) + p_N^2 \right) \\
= \frac{1}{9\beta} \left( (\alpha - c) - p_N \right)^2
\]

Network profit is equal to the network’s revenue, price times quantity, leading to Eq.11.

\[
Y_N = p_N q_N \\
= p_N \frac{2(\alpha - p_N - c)}{3\beta}
\]

Consumer surplus is the area bounded by a triangle with vertices at \(\alpha\), \(p\), and \(q_N\) (see Figure 4).

\[
Y_{CS} = \frac{1}{2}(\alpha - p)q_N \\
= \frac{1}{3} \left( \alpha - \frac{\alpha}{3} - \frac{2(p_N + c)}{3} \right) \frac{2}{3\beta} (\alpha - p_N - c) \\
= \frac{2}{9\beta} \left( \alpha^2 - 2\alpha c + c^2 + 2p_N(c - \alpha) + p_N^2 \right) \\
= \frac{2}{9\beta} \left( (\alpha - c) - p_N \right)^2
\]

Appendix C - Critical points

The network (i.e. total) quantity sold at the critical network price is found by substituting Eq.16 in Eq.8. As noted in Section 3 above, this will be the quantity corresponding to maximum welfare provided \(\chi + \psi < \frac{3}{4}\).

\[
q_N = \frac{2(\alpha - c)}{3\beta} - \frac{2(\alpha - c) \cdot 5(\chi + \psi) - 3}{2 \times 3\beta \cdot 4(\chi + \psi) - 3} \\
= \frac{(\alpha - c)}{3\beta} \left( \frac{2 - \frac{5(\chi + \psi) - 3}{4(\chi + \psi) - 3}}{2} \right)
\]

Inspection of this equation reveals that the quantity goes to zero (a minimum as expected) when \(\chi + \psi\) goes to the limit of 1.
The total welfare - which will be a maximum or minimum - at the critical network price is found by rearranging Eq.13,

\[ Y = \frac{2}{9\beta} \left[ p_N^2 (4(\chi + \psi) - 3) - p_N (\alpha - c)(5(\chi + \psi) - 3) \right] \]

\[ + \frac{2}{9\beta} [(\chi + \psi)(\alpha - c)^2] \]

and then substituting the critical \( p_N \) of Eq.16:

\[ Y = \frac{2}{9\beta} \left[ \frac{(\alpha - c)^2 (5(\chi + \psi) - 3)^2}{4(4(\chi + \psi) - 3)^2} - \frac{(\alpha - c)^2 (5(\chi + \psi) - 3)^2}{2(4(\chi + \psi) - 3)} + (\alpha - c)^2(\chi + \psi) \right] \]

\[ = \frac{2(\alpha - c)^2}{9\beta} \left[ (\chi + \psi) - \frac{(5(\chi + \psi) - 3)^2}{4(4(\chi + \psi) - 3)} \right] \]

References


